7.4 Solving Polynomial Equations in Factored Form

Essential Question How can you solve a polynomial equation?

EXPLORATION 1 Matching Equivalent Forms of an Equation

Work with a partner. An equation is considered to be in *factored form* when the product of the factors is equal to 0. Match each factored form of the equation with its equivalent standard form and nonstandard form.

	Factored Form		Standard Form		Nonstandard Form
a.	$(x-1)(x-3)=0$	**A.**	$x^2-x-2=0$	**1.**	$x^2-5x=-6$
b.	$(x-2)(x-3)=0$	**B.**	$x^2+x-2=0$	**2.**	$(x-1)^2=4$
c.	$(x+1)(x-2)=0$	**C.**	$x^2-4x+3=0$	**3.**	$x^2-x=2$
d.	$(x-1)(x+2)=0$	**D.**	$x^2-5x+6=0$	**4.**	$x(x+1)=2$
e.	$(x+1)(x-3)=0$	**E.**	$x^2-2x-3=0$	**5.**	$x^2-4x=-3$

USING TOOLS STRATEGICALLY

To be proficient in math, you need to consider using tools such as a table or a spreadsheet to organize your results.

EXPLORATION 2 Writing a Conjecture

Work with a partner. Substitute 1, 2, 3, 4, 5, and 6 for x in each equation and determine whether the equation is true. Organize your results in a table. Write a conjecture describing what you discovered.

a. $(x-1)(x-2)=0$

b. $(x-2)(x-3)=0$

c. $(x-3)(x-4)=0$

d. $(x-4)(x-5)=0$

e. $(x-5)(x-6)=0$

f. $(x-6)(x-1)=0$

EXPLORATION 3 Special Properties of 0 and 1

Work with a partner. The numbers 0 and 1 have special properties that are shared by no other numbers. For each of the following, decide whether the property is true for 0, 1, both, or neither. Explain your reasoning.

a. When you add ____ to a number n, you get n.

b. If the product of two numbers is ____, then at least one of the numbers is 0.

c. The square of ____ is equal to itself.

d. When you multiply a number n by ____, you get n.

e. When you multiply a number n by ____, you get 0.

f. The opposite of ____ is equal to itself.

Communicate Your Answer

4. How can you solve a polynomial equation?

5. One of the properties in Exploration 3 is called the Zero-Product Property. It is one of the most important properties in all of algebra. Which property is it? Why do you think it is called the Zero-Product Property? Explain how it is used in algebra and why it is so important.

7.4 Lesson

What You Will Learn

- Use the Zero-Product Property.
- Factor polynomials using the GCF.
- Use the Zero-Product Property to solve real-life problems.

Core Vocabulary

factored form, *p. 378*
Zero-Product Property, *p. 378*
roots, *p. 378*
repeated roots, *p. 379*

Previous
polynomial
standard form
greatest common factor (GCF)
monomial

Using the Zero-Product Property

A polynomial is in **factored form** when it is written as a product of factors.

Standard form	Factored form
$x^2 + 2x$	$x(x + 2)$
$x^2 + 5x - 24$	$(x - 3)(x + 8)$

When one side of an equation is a polynomial in factored form and the other side is 0, use the **Zero-Product Property** to solve the polynomial equation. The solutions of a polynomial equation are also called **roots**.

Core Concept

Zero-Product Property

Words If the product of two real numbers is 0, then at least one of the numbers is 0.

Algebra If a and b are real numbers and $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 1 Solving Polynomial Equations

Solve each equation.

a. $2x(x - 4) = 0$ **b.** $(x - 3)(x - 9) = 0$

SOLUTION

a. $2x(x - 4) = 0$ Write equation.

$2x = 0 \quad or \quad x - 4 = 0$ Zero-Product Property

$x = 0 \quad or \quad x = 4$ Solve for x.

▶ The roots are $x = 0$ and $x = 4$.

b. $(x - 3)(x - 9) = 0$ Write equation.

$x - 3 = 0 \quad or \quad x - 9 = 0$ Zero-Product Property

$x = 3 \quad or \quad x = 9$ Solve for x.

▶ The roots are $x = 3$ and $x = 9$.

Check

To check the solutions of Example 1(a), substitute each solution in the original equation.

$2(0)(0 - 4) \stackrel{?}{=} 0$

$0(-4) \stackrel{?}{=} 0$

$0 = 0$ ✓

$2(4)(4 - 4) \stackrel{?}{=} 0$

$8(0) \stackrel{?}{=} 0$

$0 = 0$ ✓

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Solve the equation. Check your solutions.

1. $x(x - 1) = 0$
2. $3t(t + 2) = 0$
3. $(z - 4)(z - 6) = 0$

When two or more roots of an equation are the same number, the equation has **repeated roots**.

EXAMPLE 2 **Solving Polynomial Equations**

Solve each equation.

a. $(2x + 7)(2x - 7) = 0$ **b.** $(x - 1)^2 = 0$ **c.** $(x + 1)(x - 3)(x - 2) = 0$

SOLUTION

a. $(2x + 7)(2x - 7) = 0$ — Write equation.

$2x + 7 = 0$ *or* $2x - 7 = 0$ — Zero-Product Property

$x = -\frac{7}{2}$ *or* $x = \frac{7}{2}$ — Solve for x.

▶ The roots are $x = -\frac{7}{2}$ and $x = \frac{7}{2}$.

b. $(x - 1)^2 = 0$ — Write equation.

$(x - 1)(x - 1) = 0$ — Expand equation.

$x - 1 = 0$ *or* $x - 1 = 0$ — Zero-Product Property

$x = 1$ *or* $x = 1$ — Solve for x.

▶ The equation has repeated roots of $x = 1$.

STUDY TIP

You can extend the Zero-Product Property to products of more than two real numbers.

c. $(x + 1)(x - 3)(x - 2) = 0$ — Write equation.

$x + 1 = 0$ *or* $x - 3 = 0$ *or* $x - 2 = 0$ — Zero-Product Property

$x = -1$ *or* $x = 3$ *or* $x = 2$ — Solve for x.

▶ The roots are $x = -1$, $x = 3$, and $x = 2$.

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Solve the equation. Check your solutions.

4. $(3s + 5)(5s + 8) = 0$ **5.** $(b + 7)^2 = 0$ **6.** $(d - 2)(d + 6)(d + 8) = 0$

Factoring Polynomials Using the GCF

To solve a polynomial equation using the Zero-Product Property, you may need to *factor* the polynomial, or write it as a product of other polynomials. Look for the *greatest common factor* (GCF) of the terms of the polynomial. This is a monomial that divides evenly into each term.

EXAMPLE 3 **Finding the Greatest Common Monomial Factor**

Factor out the greatest common monomial factor from $4x^4 + 24x^3$.

SOLUTION

The GCF of 4 and 24 is 4. The GCF of x^4 and x^3 is x^3. So, the greatest common monomial factor of the terms is $4x^3$.

▶ So, $4x^4 + 24x^3 = 4x^3(x + 6)$.

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7. Factor out the greatest common monomial factor from $8y^2 - 24y$.

EXAMPLE 4 Solving Equations by Factoring

Solve (a) $2x^2 + 8x = 0$ and (b) $6n^2 = 15n$.

SOLUTION

a.

$2x^2 + 8x = 0$	Write equation.
$2x(x + 4) = 0$	Factor left side.
$2x = 0$ *or* $x + 4 = 0$	Zero-Product Property
$x = 0$ *or* $x = -4$	Solve for x.

▶ The roots are $x = 0$ and $x = -4$.

b.

$6n^2 = 15n$	Write equation.
$6n^2 - 15n = 0$	Subtract $15n$ from each side.
$3n(2n - 5) = 0$	Factor left side.
$3n = 0$ *or* $2n - 5 = 0$	Zero-Product Property
$n = 0$ *or* $n = \frac{5}{2}$	Solve for n.

▶ The roots are $n = 0$ and $n = \frac{5}{2}$.

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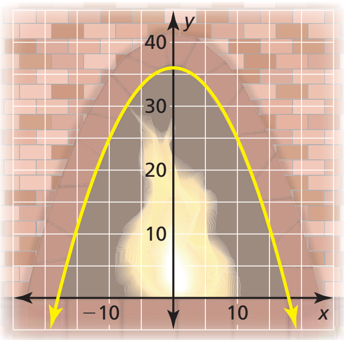
Solve the equation. Check your solutions.

8. $a^2 + 5a = 0$ **9.** $3s^2 - 9s = 0$ **10.** $4x^2 = 2x$

Solving Real-Life Problems

EXAMPLE 5 Modeling with Mathematics

You can model the arch of a fireplace using the equation $y = -\frac{1}{9}(x + 18)(x - 18)$, where x and y are measured in inches. The x-axis represents the floor. Find the width of the arch at floor level.



SOLUTION

Use the x-coordinates of the points where the arch meets the floor to find the width. At floor level, $y = 0$. So, substitute 0 for y and solve for x.

$y = -\frac{1}{9}(x + 18)(x - 18)$	Write equation.
$0 = -\frac{1}{9}(x + 18)(x - 18)$	Substitute 0 for y.
$0 = (x + 18)(x - 18)$	Multiply each side by -9.
$x + 18 = 0$ *or* $x - 18 = 0$	Zero-Product Property
$x = -18$ *or* $x = 18$	Solve for x.

The width is the distance between the x-coordinates, -18 and 18.

▶ So, the width of the arch at floor level is $|-18 - 18| = 36$ inches.

Monitoring Progress

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11. You can model the entrance to a mine shaft using the equation $y = -\frac{1}{2}(x + 4)(x - 4)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the entrance at ground level.

7.4 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

Vocabulary and Core Concept Check

1. **WRITING** Explain how to use the Zero-Product Property to find the solutions of the equation $3x(x - 6) = 0$.

2. **DIFFERENT WORDS, SAME QUESTION** Which is different? Find *both* answers.

Solve the equation $(2k + 4)(k - 3) = 0$.	Find the values of k for which $2k + 4 = 0$ *or* $k - 3 = 0$.
Find the value of k for which $(2k + 4) + (k - 3) = 0$.	Find the roots of the equation $(2k + 4)(k - 3) = 0$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–8, solve the equation. *(See Example 1.)*

3. $x(x + 7) = 0$
4. $r(r - 10) = 0$
5. $12t(t - 5) = 0$
6. $-2v(v + 1) = 0$
7. $(s - 9)(s - 1) = 0$
8. $(y + 2)(y - 6) = 0$

In Exercises 9–20, solve the equation. *(See Example 2.)*

9. $(2a - 6)(3a + 15) = 0$
10. $(4q + 3)(q + 2) = 0$
11. $(5m + 4)^2 = 0$
12. $(h - 8)^2 = 0$
13. $(3 - 2g)(7 - g) = 0$
14. $(2 - 4d)(2 + 4d) = 0$
15. $z(z + 2)(z - 1) = 0$
16. $5p(2p - 3)(p + 7) = 0$
17. $(r - 4)^2(r + 8) = 0$
18. $w(w - 6)^2 = 0$
19. $(15 - 5c)(5c + 5)(-c + 6) = 0$
20. $(2 - n)\left(6 + \frac{2}{3}n\right)(n - 2) = 0$

In Exercises 21–24, find the x-coordinates of the points where the graph crosses the x-axis.

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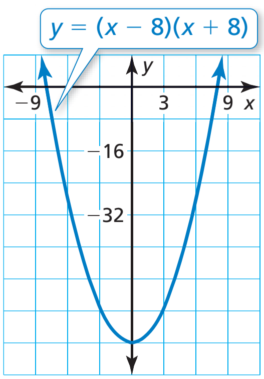

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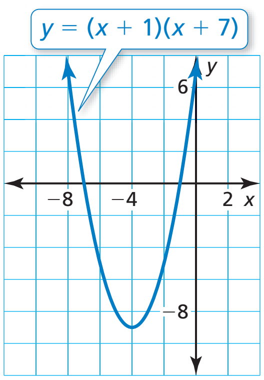

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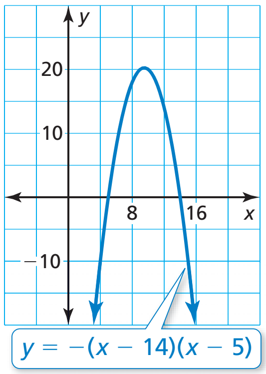

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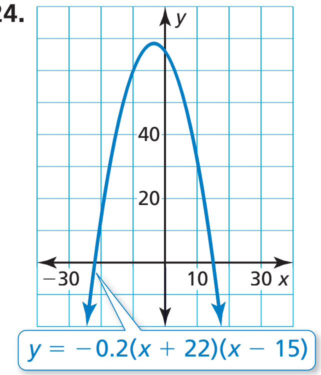


In Exercises 25–30, factor the polynomial. *(See Example 3.)*

25. $5z^2 + 45z$
26. $6d^2 - 21d$
27. $3y^3 - 9y^2$
28. $20x^3 + 30x^2$
29. $5n^6 + 2n^5$
30. $12a^4 + 8a$

In Exercises 31–36, solve the equation. *(See Example 4.)*

31. $4p^2 - p = 0$
32. $6m^2 + 12m = 0$
33. $25c + 10c^2 = 0$
34. $18q - 2q^2 = 0$
35. $3n^2 = 9n$
36. $-28r = 4r^2$

37. **ERROR ANALYSIS** Describe and correct the error in solving the equation.

✗

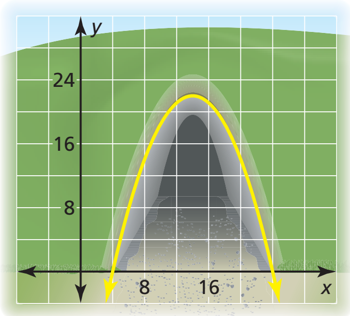
$$6x(x + 5) = 0$$
$$x + 5 = 0$$
$$x = -5$$

The root is $x = -5$.

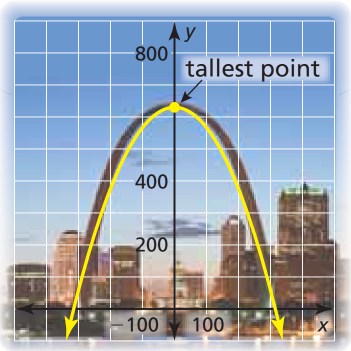
38. ERROR ANALYSIS Describe and correct the error in solving the equation.

> ✗
> $3y^2 = 21y$
> $3y = 21$
> $y = 7$
> The root is $y = 7$.

39. MODELING WITH MATHEMATICS The entrance of a tunnel can be modeled by $y = -\frac{11}{50}(x - 4)(x - 24)$, where x and y are measured in feet. The x-axis represents the ground. Find the width of the tunnel at ground level. *(See Example 5.)*



40. MODELING WITH MATHEMATICS The Gateway Arch in St. Louis can be modeled by $y = -\frac{2}{315}(x + 315)(x - 315)$, where x and y are measured in feet. The x-axis represents the ground.

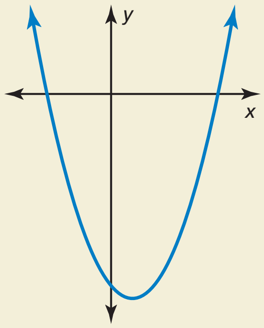


a. Find the width of the arch at ground level.

b. How tall is the arch?

41. MODELING WITH MATHEMATICS A penguin leaps out of the water while swimming. This action is called porpoising. The height y (in feet) of a porpoising penguin can be modeled by $y = -16x^2 + 4.8x$, where x is the time (in seconds) since the penguin leaped out of the water. Find the roots of the equation when $y = 0$. Explain what the roots mean in this situation.

42. HOW DO YOU SEE IT? Use the graph to fill in each blank in the equation with the symbol + or −. Explain your reasoning.



$y = (x \;\_\_\; 5)(x \;\_\_\; 3)$

43. CRITICAL THINKING How many x-intercepts does the graph of $y = (2x + 5)(x - 9)^2$ have? Explain.

44. MAKING AN ARGUMENT Your friend says that the graph of the equation $y = (x - a)(x - b)$ always has two x-intercepts for any values of a and b. Is your friend correct? Explain.

45. CRITICAL THINKING Does the equation $(x^2 + 3)(x^4 + 1) = 0$ have any real roots? Explain.

46. THOUGHT PROVOKING Write a polynomial equation of degree 4 whose only roots are $x = 1$, $x = 2$, and $x = 3$.

47. REASONING Find the values of x in terms of y that are solutions of each equation.

a. $(x + y)(2x - y) = 0$

b. $(x^2 - y^2)(4x + 16y) = 0$

48. PROBLEM SOLVING Solve the equation $(4^{x-5} - 16)(3^x - 81) = 0$.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

List the factor pairs of the number. *(Skills Review Handbook)*

49. 10

50. 18

51. 30

52. 48